

6. A coordinate system (u,v,w) has an origin at $(4,5,5)$, the u -axis has a direction vector of $(1,4,8)/9$, the v axis has a direction vector of $(4,7,-4)/9$, and the w -axis has a direction vector of $(-8,4,-1)/9$.
- a. (5 points) What are the (u,v,w) coordinates of a point whose (x,y,z) coordinates are $(13,14,23)$?

- b. (5 points) Write down the 4×4 matrix M that transforms a point (x, y, z) into a point (u, v, w) . That is, find M such that

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}.$$

7. (5 points) Recall that a window is a rectangle that encloses the objects you want to view. It can be defined in terms of the center of the window (W_x, W_y) and its width and height (W_x, W_y) . A viewport is a rectangle in pixel coordinates that indicates where you want the objects inside the window to be drawn. It, too, is specified in terms of a center (V_x, V_y) and width and height (V_x, V_y) . A window-to-viewport mapping is a linear transformation that transforms each corner of the window to its corresponding corner of the viewport. Find the 3×3 matrix that specifies the window-to-viewport mapping for the window $W_x=4, W_y=3, W_x=5, W_y=5$ and the viewport $V_x=350, V_y=400, V_x=250, V_y=250$.

8. (6 points) A point light source is located at $(10, 20, 20)$. A flat mirror has a normal vector $(.6, 0, .8)$ and the point $\mathbf{P} = (10, 14, 12)$ lies on the mirror. What is the direction at which the light reflects off of the mirror at point \mathbf{P} ?
9. (6 points) What is the unit normal vector for the triangle with vertices $(1,2,3)$, $(3,2,1)$ and $(2,3,-1)$?
10. (6 points) What is the 3×3 transformation matrix for performing a 2D rotation of 90° about the point $(3, 3)$?

11. (8 points) What are the Cartesian coordinates at which the ray $(16, 4, 3) + (-3, 0, 1)t$ intersects the sphere of radius 9, centered at the origin?

12. (10 points) In Figure 1, if $\mathbf{E} = (4, 14, 7)$, $\mathbf{A} = (1, 2, 3)$, and $\mathbf{U}_p = (0, 1, 0)$, find the unit vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} .

$\mathbf{u} =$ _____ $\mathbf{v} =$ _____ $\mathbf{w} =$ _____

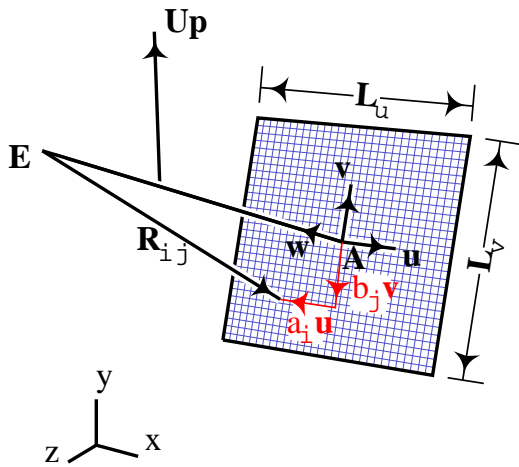
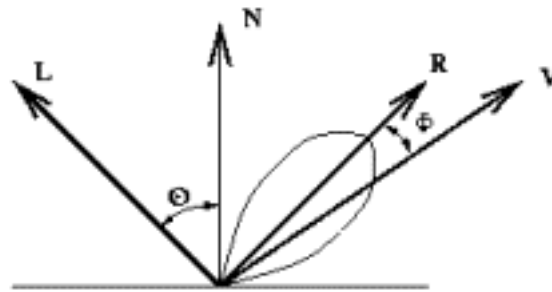


Figure 1: Eye coordinate system

13. (10 points) A triangle is being rasterized. Its three vertices are located in the center of pixels (300,300), (100,100), and (400,200). The z -coordinates of those vertices, for the purpose of z -buffering, are 15, 25, and 40 respectively. What is the z coordinate of the pixel (300,200)?



Distribution of Scattered Reflection

14. (9 points)
 Recall these equations for a simplified Phong lighting model.

$$r' = c_a r + (1 - c_a) c_s$$

$$g' = c_a g + (1 - c_a) c_s$$

$$b' = c_a b + (1 - c_a) c_s$$

where

$$R = 2(L \cdot N)N - L$$

$$c_a = a + (1 - a)L \cdot N$$

$$c_s = (R \cdot V)^n$$

In this figure, N is the surface normal, L is the unit vector to the light, R is the reflection of L , and V is the unit vector from the point to the camera.

- What does the quantity a represent?
- What does c_s represent?
- If n is increased, does the surface look more shiny or less shiny?