

7:00a-10:00a 25 April 2002, 3710 HBLL

Closed book. Calculator allowed. 150 points.

Give complete answers and show all work for mathematical questions!

1. (5 points) Imagine that you want to use graphics hardware to simulate reflection off of a shiny curved surface, so that the reflection changes in real-time if the curved object is rotated or translated.

a. What technique would you use?

b. Describe briefly how that technique works.

c. What is the main advantage and disadvantage of that technique compared to ray tracing?

2. (2 points) What does the acronym CAD stand for? _____

3. (4 points) What advantage does Bresenham's line-drawing algorithm have over the DDA algorithm and when would that advantage be significant?

4. (2 points) Name three movies that were made using computer graphics.

5. (4 points) A cubic Bezier curve has control points $(100,200)$, $(180,352)$, $(240,400)$, $(300,350)$ where the Cartesian coordinates are given in terms of screen space. What step size (that is, Δt) should be used in determining which pixels should be turned on for that curve?

6. (3 points) What is the cosine of the angle between the vectors $(3, 4, 12)$ and $(12, 4, -3)$? (express the answer as a fraction)

7. (3 points) How long is the projection of the vector $(-1, 1, 1)$ onto the vector $(3, 4, 12)$?

8. (3 points) In the 1970s, why were scan-line hidden surface algorithms more popular than z-buffer hidden surface algorithms?

9. (5 points) Create a single 3×3 transformation matrix that has the effect of performing the following two operations: First, scale by (2,3). Second, rotate 90° clockwise.

10. (5 points) Create a single 3×3 transformation matrix that has the effect of performing a window-viewport mapping. The window has lower-left corner (3,4) and upper-right corner (5,6) and the viewport has lower-left corner (1,2) and upper-right corner (21,22). $(x_w, y_w, 1)$ is a point in the window, and $(x_v, y_v, 1)$ is the coordinates of that point mapped to the viewport.

$$\begin{Bmatrix} x_v \\ y_v \\ 1 \end{Bmatrix} = \begin{bmatrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{bmatrix} \begin{Bmatrix} x_w \\ y_w \\ 1 \end{Bmatrix}$$

11. (5 points) There is only one true normal vector for any single triangle. Yet, often in computer graphics a *different* normal vector is specified for each vertex of a triangle. Why is that so?

Describe a method for determining those different normal vectors.

12. (5 points) If a line segment with endpoints (3,4) and (7,9) is drawn using the DDA algorithm, which pixels will be lit? Assume that integer coordinates refer to pixel centers.

13. (2 points) An array of pixels is called a _____.
14. (2 points) The *depth* of a frame buffer means _____.
15. (2 points) The process of computing which pixels should be turned on to draw a given line or triangle is called _____.
16. (2 points) Forward differencing is a fast way to draw curves; the main drawback of forward differencing is _____.

Multiple Choice. Circle the correct answer. 3 points each.

17. Aspect ratio is
- a The width of a rectangle divided by its height.
 - b The largest barycentric coordinate divided by the smallest.
 - c The window area divided by the viewport area.
 - d The number of pixels that make up the letter 'A' in a bitmap font.
18. Bump mapping
- a is synonymous with texture mapping.
 - b is another name for texture mapping but when the image used in the texture map looks “bumpy.”
 - c is achieved by actually perturbing an otherwise smooth surface, using a “displacement map.”
 - d is achieved by perturbing the normal vectors.
19. In an interlaced CRT operating at 60 Hz,
- a each row is refreshed 60 times per second.
 - b each row is refreshed 30 times per second.
 - c each row is refreshed 120 times per second.
20. Circle the false statement:
- a The painter’s algorithm requires you to store the z-coordinate of every pixel.
 - b Back-face culling can speed up rendering, even in hardware.
 - c The Z-buffer algorithm requires all triangles to be rasterized.
21. A point light source is defined to a a light source that
- a is always located at the eye.
 - b is always located at infinity.
 - c always emits light with constant intensity in all directions.
 - d always emits pure white light — $(r,g,b) = (1,1,1)$.

22. An orthographic projection is a view in which

- a the eye is located at an infinite distance away from the lookat point.
- b the eye is located at a point light source.
- c the view angle is 45° or greater.
- d the aspect ratio is 1.

23. A perfectly diffuse surface

- a is translucent but not transparent.
- b has dull but detectable highlights in the presence of point light sources.
- c is also called an ambient surface.
- d is also called a Lambertian surface.

24. Which of the following statements about the human visual system is false?

- a There are three types of rods.
- b The iris controls how much light enters the eye.
- c Red light and green light of the same intensity appear to us to have different brightness.
- d The color of light is a function of its wavelength.

25. A color lookup table

- a is primarily used to adjust for non-linearities in the CRT.
- b is primarily used when a frame buffer is not very deep.
- c is primarily used to speed up the calculation of colors inside of a shadow.
- d is primarily used to speed up the calculation of colors near highlights.

26. A display list

- a is used in retained-mode graphics.
- b is a two-dimensional array of pixels.
- c is a linked list of 4×4 transformation matrices to be concatenated.
- d is used in immediate graphics mode.

27. Circle True or False (2 points each)

- T F In the Phong lighting model, a larger exponent suggests a more shiny surface.
- T F Ambient light increases in intensity as an object moves closer to the light source.
- T F In computer graphics, "clipping" refers to what happens when you have more triangles than your graphics card can render in $1/30^{th}$ of a second.
- T F Three-dimensional translations can be represented using a 3×3 matrix, but rotations require a 4×4 matrix.

28. (3 points) What is the unit normal vector for the triangle with vertices $(1,2,3)$, $(3,2,1)$ and $(2,3,1)$?

29. (3 points) What is the equation of the plane that contains the triangle with vertices $(1,2,3)$, $(3,2,1)$ and $(2,3,1)$?

30. (3 points) What is the area of the triangle with vertices $(1,2,3)$, $(3,2,1)$ and $(2,3,1)$?

31. (6 points) The 3DDDA method for speeding up ray tracing is a lot like a hash table. Using that analogy, what are the hash table “buckets” in the 3DDDA algorithm?

What is contained in the linked list of each bucket?

What significant differences are there between a hash table and a 3DDDA?

32. (5 points) For a degree 3 polynomial, $f(0) = 1$, $f(1) = 3$, $f(2) = 1$, $f(3) = 2$. Find $f(4)$ using forward differencing.

33. (10 points) In Figure 1, if $\mathbf{E} = (5, 5, 15)$, $\mathbf{A} = (1, 2, 3)$, and $\mathbf{Up} = (4, 3, 0)$, find the unit vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} .

$\mathbf{u} =$ _____ $\mathbf{v} =$ _____ $\mathbf{w} =$ _____

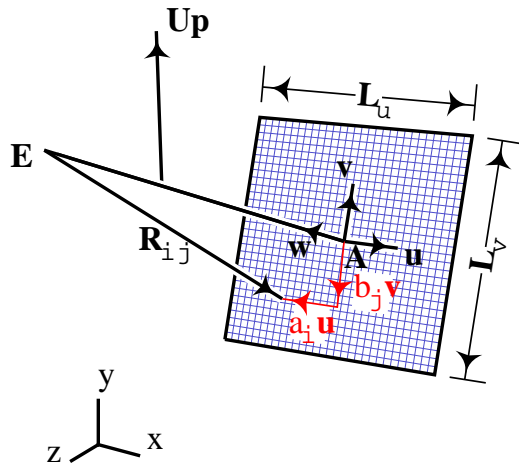


Figure 1. Eye coordinate system.

34. (6 points) This problem deals with how to express coordinate system transformations in matrix form. Imagine that in Figure 1, $\mathbf{u} = (\frac{12}{13}, -\frac{3}{13}, -\frac{4}{13})$, $\mathbf{v} = (\frac{4}{13}, \frac{12}{13}, \frac{3}{13})$, and $\mathbf{w} = (\frac{3}{13}, -\frac{4}{13}, \frac{12}{13})$ and $\mathbf{A} = (0, 1, 2)$. Given a point in world coordinates $\mathbf{P} = (x, y, z, 1)$, fill in the 16 elements of this 4×4 matrix so that the matrix multiplication will correctly compute the eye coordinates $(u, v, w, 1)$ of the point \mathbf{P} .

$$\begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix} = \begin{bmatrix} \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

35. (6 points) If in Figure 1, $\mathbf{E} = (0, 0, 8)$, $\mathbf{A} = (0, 0, 0)$, $\mathbf{Up} = (0, 1, 0)$, $\mathbf{u} = (1, 0, 0)$, $\mathbf{v} = (0, 1, 0)$, $\mathbf{w} = (0, 0, 1)$, $L_u = L_v = 5$, and the number of pixels is 500×500 . A triangle vertex has world coordinates $(1, 2, 4)$. What pixel will that triangle vertex get mapped to?

36. (5 points) The operation of rotating by an angle of 70° about an axis that has a direction vector of $(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$ and that passes through the point $(0, 0, 4)$ can be performed using a single 4×4 matrix that is a concatenation of five elementary 4×4 matrices: $M_1M_2M_3M_4M_5$. Describe precisely in words what those five matrices do (such as “translate by $(1,2,3)$ ” or “rotate 90° about the x -axis. You do *not* need to show the contents of the matrices. The order in which the concatenation occurs is $M_1M_2M_3M_4M_5$.

M_1 : _____

M_2 : _____

M_3 : _____

M_4 : _____

M_5 : _____

37. (5 points) A point light source is located at $(15, 19, 15)$. A flat mirror has a normal vector $(2, 6, 3)$ and the point $\mathbf{P} = (10, 14, 12)$ lies on the mirror. What is the direction at which the light reflects off of the mirror at point \mathbf{P} ?

38. (6 points) A triangle has vertices $P_s = (0, 2, 3)$, $P_t = (3, 3, 4)$, and $P_u = (2, 6, 7)$. What are the barycentric coordinates of the point $(2, 3, 4)$?