2:30p-5:30p 25 April 2002, 369 CB
Closed book. Calculator allowed. 150 points.
Give complete answers and show all work for mathematical questions!

1. (4 points) A cubic Bézier curve has control points (100,100), (300,300), (500,300), (700,500) where the Cartesian coordinates are given in terms of pixel coordinates. What step size (that is, Δt) should be used in determining which pixels should be turned on for that curve?

2. (3 points) What is the cosine of the angle between the vectors $(1, 2, 2)$ and $(12, 4, -3)$? (express the answer as a fraction)

3. (2 points) Forward differencing is a fast way to draw curves; the main drawback of forward differencing is ______________________________.

4. (3 points) In which computer graphics operation is the "mod" function used? a. Polygon clipping b. Perspective transformation c. Area filling using patterns d. Phong shading

5. (3 points) Explain how the Bresenham algorithm is related to forward differencing.

6. (4 points) Suppose a degree three Bézier curve $\mathbf{P}(t)$ is to be drawn using forward differencing. If

$$\mathbf{P}(0) = (0, 0), \quad \mathbf{P}(0.01) = (1, 0), \quad \mathbf{P}(0.02) = (2, 1), \quad \mathbf{P}(0.03) = (3, 2)$$

find $\mathbf{P}(0.04)$ using forward differencing.

7. (3 points) What is the length of the projection of the vector $(2, 3, 1)$ onto the vector $(4, 2, 4)$?

8. (4 points) In the real world, light rays begin at light sources, bounce off one or more objects, then entire our eye. Why in ray tracing do we typically have rays originate at the eye position rather than from light sources?

9. (2 points) What type of geometric primitive is the "Utah Teapot" modelled using?

10. (6 points) Create a single 3×3 transformation matrix that has the effect of rotating $90°$ clockwise about the point $(2, 3)$.

11. (4 points) Why does OpenGL allow you to specify a different normal vector for each vertex of a triangle?

12. (2 points) The *depth* of a frame buffer means ______________________________.
13. (2 points) The process of computing which pixels should be turned on to draw a given line or triangle is called ____________________.
14. (5 points) Imagine that you want to use graphics hardware to simulate reflection off of a shiny curved surface, so that the reflection changes in real-time if the curved object is rotated or translated.
a. What technique would you use?

b. Describe briefly how that technique works.

c. What is the main advantage and disadvantage of that technique compared to ray tracing?

15. (6 points) Create a single 3×3 transformation matrix that has the effect of performing a window-viewport mapping. The window has lower-left corner $(30, 40)$ and upper-right corner $(62, 72)$ and the viewport has lower-left corner $(0, 0)$ and upper-right corner $(256, 256)$. $(x_w, y_w, 1)$ is a point in the window, and $(x_v, y_v, 1)$ are the coordinates of that point mapped to the viewport.

$$\begin{Bmatrix} x_v \\ y_v \\ 1 \end{Bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix} \begin{Bmatrix} x_w \\ y_w \\ 1 \end{Bmatrix}$$

Multiple Choice. Circle the correct answer. 3 points each.

16. The aspect ratio of a viewport that is 200 pixels wide and 100 pixel tall is

a. 2 **b.** 200 **c.** $\frac{1}{2}$ **d.** It depends on where the lower-left corner is.

17. Bump mapping

a is synonymous with environment mapping.

b is a special case of texture mapping in which the image used in the texture map looks "bumpy."

c is used to determine normal vectors.

d is achieved by actually perturbing an otherwise smooth surface, using a "displacement map."

18. A point light source is defined to be a light source that

a simulates a laser, in that it never diverges..

b is physically represented as a single electric light bulb suspended by a wire.

c emits light with constant intensity in all directions.

d always emits pure white light— (r,g,b) = (1,1,1).

19. A perfectly diffuse surface

a is everywhere the same color.

b has dull but detectable highlights in the presence of point light sources.

c is also called an ambient surface.

d is a surface for which each point reflects light with the same intensity in all directions that are visible from the point.

20. Circle True or False (2 points each)

T F In the Gouraud lighting model, a larger exponent suggests a more shiny surface.

T F Ambient light increases in intensity as an object moves closer to the light source.

T F In order to represent translations in matrix form, it is necessary to use homogeneous coordinates (x,y,z,1).

21. (4 points) What is the unit normal vector for the triangle with vertices (6,2,0), (0,3,2), and (8,0,1)?

22. (3 points) What is the equation of the plane that contains the triangle with vertices (6,2,0), (0,3,2), and (8,0,1)?

23. (3 points) What is the area of the triangle with vertices (6,2,0), (0,3,2), and (8,0,1)?

24. (5 points) Two billiard balls each have a radius of one inch. At time $t = 0$, their respective positions on the billiard table (Cartesian coordinates given in inches) are $\mathbf{B}_1 = (0, 0)$ and $\mathbf{B}_2 = (43.2, 7.6)$ and their respective velocities in inches per second are $\mathbf{v}_1 = (3, 4)$ and $\mathbf{v}_2 = (-4, 3)$. Compute the time t at which the two balls collide.

25. (5 points) Assuming that the two balls in the preceding problem have the same mass, what is the new velocity of each ball after they collide?

26. (6 points) A triangle has vertices at pixels $P_1 = (40, 50)$, $P_2 = (10, 20)$, and $P_3 = (70, 35)$. The color of pixel P_1 is $(r, g, b) = (.7, .3, .5)$, of pixel P_2 is $(r, g, b) = (.4, .9, .8)$, and of pixel P_3 is $(r, g, b) = (1, 0, .2)$. Using Gouraud shading, what is the color of pixel $(20, 30)$? (note that this pixel lies on a line from P_1 to P_2.

27. (6 points) The 3DDDA method for speeding up ray tracing is a lot like a hash table. Using that

analogy, what are the hash table "buckets" in the 3DDDA algorithm?

What is contained in the linked list of each bucket?

What significant differences are there between a hash table and a 3DDDA?

28. (8 points) In Figure 1, if $\mathbf{E} = (3, 5, 9)$, $\mathbf{A} = (1, 2, 3)$, and $\mathbf{Up} = (0, 3, 2)$, find the unit vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

$\mathbf{u}$ = ______________________ $\mathbf{v}$ = ______________________ $\mathbf{w}$ = ______________________

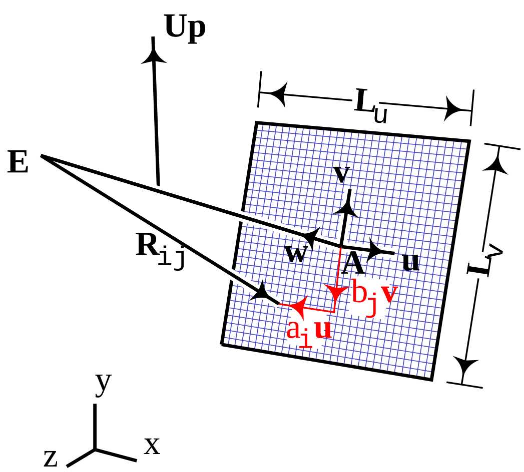

Figure 1. Eye coordinate system.

29. (4 points) For the eye coordinate system in the preceding problem, what are the eye coordinates for a point whose world coordinates are $(2, 2, 5)$?

30. (8 points) This problem deals with how to express coordinate system transformations in matrix form. Imagine that in Figure 1, $\mathbf{u} = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$, $\mathbf{v} = (\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3})$, and $\mathbf{w} = (\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$ and $\mathbf{A} = (0, 1, 2)$. Given a point in world coordinates $\mathbf{P} = (x, y, z, 1)$, fill in the 16 elements of this 4×4 matrix so that the matrix multiplication will correctly compute the eye coordinates $(u, v, w, 1)$ of the point $\mathbf{P}$.

$$\left\{ \begin{array}{c} u \\ v \\ w \\ 1 \end{array} \right\} = \left[\begin{array}{cccc} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{array} \right] \left\{ \begin{array}{c} x \\ y \\ z \\ 1 \end{array} \right\}$$

What are the eye coordinates for a point whose world coordinates are $(5, 2, 1)$?

31. (8 points) If in Figure 1, $\mathbf{E} = (0, 0, 10)$, $\mathbf{A} = (0, 0, 2)$, $\mathbf{Up} = (0, 1, 0)$, $\mathbf{u} = (1, 0, 0)$, $\mathbf{v} = (0, 1, 0)$, $\mathbf{w} = (0, 0, 1)$, $L_u = L_v = 10$, and the number of pixels is 500×500. A triangle vertex has world coordinates $(3, 5, 0)$. What pixel will that triangle vertex get mapped to?

32. (8 points) You are creating an animation of the moon rotating about the earth while the earth rotates about the sun. The sun is centered at the origin. The earth orbits around the sun in the $x - y$ plane with a circular orbit of radius R_1 and at time $t = 0$ it is located at $(R_1, 0, 0)$. The moon orbits the earth with a circular orbit of radius R_2, and at time $t = 0$ it is located at $(R_1, R_2, 0)$. The earth orbits around the sun with an angular velocity of ω_1, so that at time t it has rotated an angle of $\omega_1 t$ about the sun. Likewise, the moon orbits around the earth with an angular velocity of ω_2. Describe the sequence of matrix operations that must be performed to place the moon in its proper position and orientation at time t. Write words such as "Translate in x by R_1", or "Translate in y by R_2", or "Rotate about the z axis by $\omega_1 t$" or "Rotate about the z axis by $\omega_2 t$." The order in which the matrix multiplication occurs is: $M_1 M_2 M_3 M_4$.

M_1:____

M_2:____

M_3:____

M_4:____

33. (5 points) A point light source is located at $(19, 23, 3)$. A flat mirror has a normal vector $(4, 4, 7)$ and the point $\mathbf{P} = (10, 14, 12)$ lies on the mirror. What is the direction at which the light reflects off of the mirror at point $\mathbf{P}$?

34. (6 points) A triangle has vertices $P_s = (20, 10, 5)$, $P_t = (0, 0, 0)$, and $P_u = (10, 20, 6)$. What are the barycentric coordinates of the point $(9, 12, 4)$?