Recursive Subdivision Surfaces for Geometric Modeling

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Subdivision surfaces for character animation

GERI'S GAME, Pixar Animation Studios won the OSCAR for BEST ANIMATED SHORT FILM of 1997, Pixar Animation Studios DeRose 1998.
Subdivision surfaces for character animation

GERI'S GAME, Pixar Animation Studios

The control mesh for Geri’s head, created by digitizing a full-scale model sculpted out of clay.

DeRose 1998.
Subdivision surfaces for character animation

TOY STORY 2, Pixar Animation Studios
http://www.pixar.com/
Subdivision surfaces for character animation

A BUG'S LIFE, Pixar Animation Studios
http://www.pixar.com/
Applications: Fillet operations - Zheng

Fillet Operations

From Zheng
Applications: Zhang adaptive subdivision

Mesh 13314

Mesh 9006
Generalization of Doo-Sabin

- The idea consists of repeatedly applying the set of rules $R$ recursively to subsequently generated polyhedra:
If R is properly chosen, at infinity the sequence of polyhedra converges to a smooth surface.
Generalization of Catmull-Clark

3 iterations of Catmull-Clark Subdivision
Generalization of Catmull-Clark

3 iterations of Catmull-Clark Subdivision
Catmull-Clark Scheme

- The idea consists of repeatedly applying the set of rules R recursively to subsequently generated polyhedra:

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Part 1.
Basic Concepts and Existing Schemes
Why Subdivision?

- Arbitrary Topology
Irregular Topology
Why Subdivision?

- Arbitrary Topology
- Scalability
- Uniform of Representation
- Numerical Stability
- Code Simplicity
- Interrogation
What is a mask?

New vertex = \[
\sum_{i=0}^{n-1} \frac{m_i v_i}{\sum_{i=0}^{n-1} m_i}
\]
Subdivision Curves
Chaikin’s Curve

- Cutting off corners
Chaikin’s Curve
Chaikin’s Curve
Chaikin’s Curve
Reisenfeld and Forrest showed that the limit curve is the quadratic B-spline
Chaikin’s Curve: 2 steps
Chaikin’s Curve

Two steps

- Split: inserting midpoint
Chaikin’s Curve

Two steps

- Split: inserting midpoint
Chaikin’s Curve

Two steps
- Averaging
Chaikin’s Curve

Two steps

- Computing new vertices

$V_i^{1/4}$  $V_{i+1}^{3/4}$
Chaikin’s Curve

Two steps

- Computing new vertices

$V_i \quad 1/4 \quad V_{i+1} \quad 3/4$
Cubic B-spline Curve:
Cubic B-spline Curve:
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Cubic B-spline Curve:
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Cubic B-spline Curve:
Cubic B-spline Curve

\[ \frac{1}{2} V_{i-1} \] \[ \frac{1}{2} V_i \]
Cubic B-spline Curve:
Subdivision Surfaces
Chaikin Tensor Product

- Regular case
Chaikin Tensor Product

- Curve subdivision:
  Repeated knot insertion.
  Chaikin: insert at mid interval
Chaikin Tensor Product

- Regular case: Knot insertion at 1/2
Chaikin Tensor Product

- Regular case: Knot insertion at 1/2
Chaikin Tensor Product

- Regular case: Knot insertion at 1/2
Chaikin Tensor Product

- Regular case: Knot insertion at 1/2
Chaikin Tensor Product

- Regular case: Knot insertion at 1/2
Chaikin Tensor Product

- Original face is replaced by a F-face
Chaikin Tensor Product

- Original edge is replaced by an E-face
Chaikin Tensor Product

- Original vertex is replaced by a V-face
Catmull-Clark tensor product

Inserting knots at mid-interval in one direction
Catmull-Clark tensor product

Inserting knots at mid-interval in one direction
Catmull-Clark tensor product

Inserting knots at mid-interval in the other direction
Catmull-Clark tensor product

A new mesh is generated

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Catmull-Clark tensor product

A face generates a F-vertex

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Catmull-Clark tensor product

A face generates a F-vertex
Catmull-Clark tensor product

An edge generates a E-vertex
Catmull-Clark tensor product

An edge generates a E-vertex

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A vertex generates a V-vertex
Catmull-Clark tensor product

A vertex generates a V-vertex

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Arbitrary topology

Regular case: insert knots \((1/2,1/2)\)

What to do with arbitrary topology?
A Recursive Subdivision Surface $S$ can be defined by $S = (P_0, R)$

- $P_0$ is a polyhedral Network and $R$ is a set of rules.
Generalization of Doo-Sabin

- Start with a cube (8 points)
Generalization of Doo-Sabin

- Cut vertices and edges by plane
Generalization of Doo-Sabin

- Repeat cutting process
Generalization of Doo-Sabin

- One more time
The idea consists of repeatedly applying the set of rules R recursively to subsequently generated polyhedra:
Generalization of Doo-Sabin

- If $R$ is properly chosen, at infinity the sequence of polyhedra converges to a smooth surface.

At the limit

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Generalization of Doo-Sabin

- Doo-Sabin Quadratic:

  F-Face
Generalization of Doo-Sabin

- Doo-Sabin Quadratic:
Generalization of Doo-Sabin

- Doo-Sabin Quadratic:

- V-Face
- F-Face
- E-Face

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Generalization of Doo-Sabin

- The new vertices are computed:

$$v_i = \sum_{j=0}^{m} \alpha_{ij} v_j$$

Where

$$\alpha_{ii} = \frac{n + 5}{4n} \text{ and } \alpha_{ij} = \frac{3 + 2 \cos(2\pi (i - j) / n)}{4n}$$
Generalization of Doo-Sabin

The masks for generating new vertices are:

\[ \alpha_i = \frac{3 + 2 \cos\left(\frac{2\pi i}{n}\right)}{4n} \]

\[ \alpha_0 = \frac{n + 5}{4n} \]
Doo-Sabin Scheme

- Produces C1 surfaces everywhere.
- It is Quadratic B-spline in the regular setting.
Generalization of Catmull-Clark

- For a face generate a F-vertex

F-vertex
Generalization of Catmull-Clark

- For a face generate a F-vertex

\[ \sum_{i=0}^{n-1} v_i = 1 \]

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Generalization of Catmull-Clark

- For an edge generate a E-vertex: average of its two vertices and the F-vertices of the faces sharing the edge.

\[
\frac{1}{4} \left( \sum_{i=0}^{n-1} v_i + v_{i+1} + f_i + f_{i+1} \right)
\]

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Generalization of Catmull-Clark

- For an edge generate an E-vertex: average of its two vertices and the F-vertices of the faces sharing the edge.
Generalization of Catmull-Clark

- For a vertex generate a V-vertex: average of its F-vertices, E-vertices and the vertex itself
  - F-vertex: $\frac{1}{n^2}$
  - E-vertex: $\frac{1}{n^2}$
  - V-vertex: $\frac{n-2}{n}$

$$\frac{n-2}{n} v_j + \frac{1}{n^2} \sum_{i=0}^{n-1} v_i + \frac{1}{n^2} \sum_{i=0}^{n-1} f_i$$
Catmull-Clark: Quad case

\[ A = \frac{3}{2n^2} \]
\[ B = \frac{1}{4n^2} \]
\[ C = 1 - \frac{3}{2n^2} - \frac{1}{4n^2} \]
Generalization of Catmull-Clark

3 iterations of Catmull-Clark Subdivision
Generalization of Catmull-Clark

3 iterations of Catmull-Clark Subdivision
The idea consists of repeatedly applying the set of rules $R$ recursively to subsequently generated polyhedra:
Catmull-Clark Scheme

- The scheme (most popular) produces C2 everywhere except at the extraordinary point (C1 only).
- Ball-Storry, Peters and Reif, Zorin, Prautzsch and many others studied its analysis.
- Sabin devised a modified version that produced bounded curvature at the extraordinary points.
Non-Uniform Scheme

Sederberg et al