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A conjecture on tangent intersections of surface patches

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Abstract

This note conjectures that if two surface patches intersect with G^1 continuity along an entire curve, the probability is one that the curve is rational. This idea has significance for surface intersection algorithms.

Keywords: Surface–surface intersections; Genus

Surface intersection algorithms are typically designed to deal with G^0 intersections, in which the normal vectors of the two surfaces are parallel at only a small number (typically zero) of points along the curve of intersection. If a connected component of an intersection curve is such that the normal vectors of the two surfaces are parallel at each point along the connected component, we will call that connected component a G^1 or tangent intersection component.

Surface intersection curves are generally quite complicated. For example, the curve of intersection between two generic bicubic patches in general position is a space curve of degree 324 and genus 433 (see (Katz and Sederberg, 1988)). The genus number is of interest primarily because only curves of genus zero can be represented in rational Bézier form. Thus, a generic intersection of two bicubic patches cannot be exactly expressed as a Bézier curve. This high genus number makes the following conjecture somewhat surprising.

Conjecture 1. Denote by $\mathbf{A} \subset R^n$ the set of all pairs of all rational surface patches of a certain fixed degree where n is the total number of coefficients in both surface patches. Thus $n = 128$ if both patches are rational bicubic, or $n = 48$ if both surfaces are total degree two rational parametric surfaces. Denote by $\mathbf{B} \subset \mathbf{A}$ the set of all pairs of such surfaces whose intersection includes a G^1 connected component

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that is genus zero, and denote by $\mathbf{C} \subset \mathbf{A}$ the set of all pairs of such surfaces whose intersection includes a G^1 connected component that is genus greater than zero (**B** and **C** are not disjoint). We conjecture that **B** and **C** are semi-algebraic sets and that the dimension of **B** is greater than the dimension of **C**.

The practical application of this conjecture, if true, is that if two parametric surface patches happen to intersect in a tangent intersection component, the probability is one that the tangent intersection component is genus zero and can therefore be expressed exactly in rational Bézier form.

While most surface intersections are G^0, G^1 intersections are not uncommon. For example, two patches that meet with C^1 or G^1 continuity along their common boundary can be thought of as exhibiting a G^1 intersection, as can a torus and a plane when the torus is laid flat against the plane. However, in each of these two examples, the curve of tangent intersection is either degree one or degree two in parameter space and is therefor genus zero.

There are special cases in which the conjecture has been proven to be true.

Case 1. One surface is a plane and the other is degree $m \times n$ parametric surface where $m \leqslant 4$ and $n \leqslant 3$.

Case 2. One surface is a quadric and the other is degree $1 \times n$ (or $n \times 1$) parametric surface.

Case 3. The two surfaces are quadrics.

Clearly, the plane and the quadrics can be represented in parametric form. However, we reason by considering these surfaces in implicit form $f(x, y, z, w) = 0$ and the second surface in parametric form $r = r(s, t)$. Then the intersection curve can be represented in (s, t) domain as $f(r(s, t)) = 0$ in which $f(r(s, t))$ is a polynomial in s and t. Decompose $f(r(s, t))$ into the product of a set of relatively prime polynomials: $f(r(s, t)) = f_1^{k_1}(s, t) f_2^{k_2}(s, t) \ldots f_m^{k_m}(s, t)$. Each $f_i(s, t) = 0$ defines a component of the intersection curve. It is easy to prove that the two surfaces are tangent along the component $f_i(s, t) = 0$ if and only if $k_i > 1$. Applying this result to the above three cases shows that for each component $f_i(s, t) = 0$ of tangency, $f_i(s, t)$ is a polynomial of degree at most 2×1, $1 \times n$, or total degree at most 2. Obviously in all these cases $f_i(s, t) = 0$ is a rational curve.

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